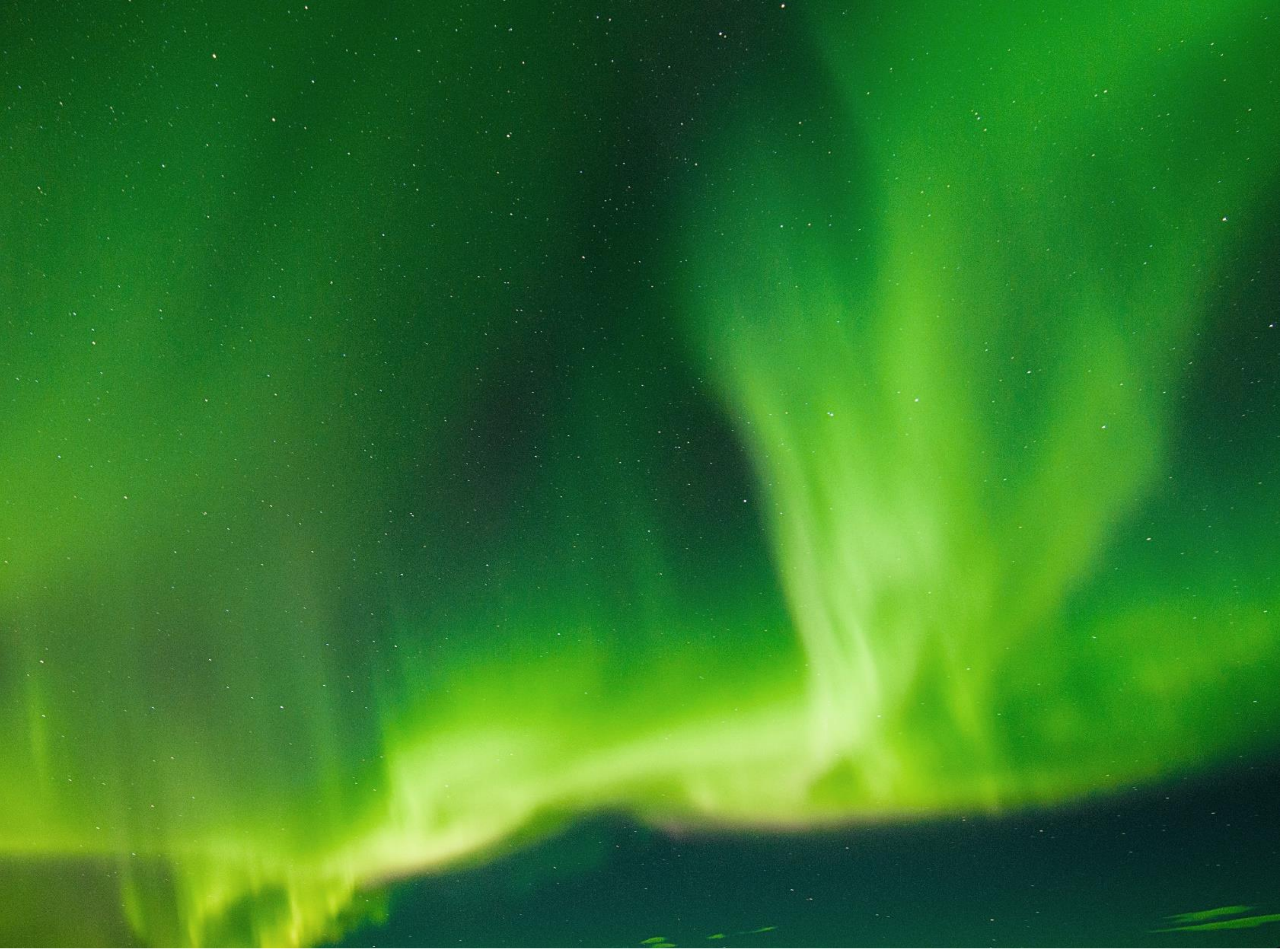

TECHNICAL NOTE

Human vision and the CIE chromaticity diagram

The CIE chromaticity diagram is instrumental in colour theory. It is widely used to describe the performances of lightings or colour cameras, and more generally any tool aimed at faithfully reproducing colours as perceived by human vision. In this document, we show a practical use of this diagram through colour reproduction of the visible spectrum.

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The CIE chromaticity diagram is instrumental in colour theory. It is widely used to describe the performances of lightings or colour cameras, and more generally any tool aimed at faithfully reproducing colours as perceived by human vision. In this document, we show a practical use of this diagram through colour reproduction of the visible spectrum.

Contents

1. Colour

The word "colour" is associated with the human perception of the visible light spectrum which covers the range of electromagnetic wavelengths from around 380 nm to about 740 nm. Admittedly, this is a somewhat rather abrupt approach to the notion of colours. Instead, we can tackle the subject starting from an experience that everyone is familiar with: Colour is most intuitively associated with the additive combination of the primary colours red, green and blue (RGB model). More technically, in terms that should sound for those familiar with image editing softwares, colour can be described in terms of **hue**, **saturation**, and **lightness** (HSL), which is an alternative representation of the RGB model. The hue value is associated with the dominant wavelength of a colour (*e.g.*, red, orange, yellow, green, cyan, blue, indigo, violet for spectral colours) or with the complementary wavelength for non-spectral colours (purple colours obtained by the combination of red and blue primaries). Saturation provides the degree of purity of a colour, *i.e.* whether it appears pale or deeply saturated. Finally, lightness quantifies the intensity of a colour in terms of darkness/lightness.

All these notions may seem rather abstract without illustrations. Fortunately, they are admirably brought together in the CIE chromaticity diagram which we will discuss in this document. Before getting into the heart of the topic, we shall first briefly introduce the photopic luminosity function (section 2) – the fundamental function of photometry science – because it is needed for the construction of the CIE chromaticity diagram.

For the sake of brevity, several complex technical concepts will be covered only succinctly in this document.

2. The photopic luminosity function

The photopic luminosity function, V(λ), quantifies the diurnal vision of the average human eye. It was adopted in 1924 by the International Commission on Illumination (CIE, French acronym for *Commission Internationale de l'Eclairage*) as the standard to define the typical spectral response of the human eye to various wavelengths of light under well-lit conditions. This function is empirical since it is based on a physiological assessment. Nevertheless, it is central in photometry as it establishes the conversion from radiometric to photometric quantities. For instance, V(λ) is used to convert radiant flux Φ_e (measured in watts, W) into luminous flux Φ_v (measured in lumens, lm). Φ_v is calculated using the following equation:

$$\Phi_v = K_{cd} \int_{\lambda} \frac{\partial \Phi_e}{\partial \lambda} V(\lambda) d\lambda \ = K_{cd} \int_{\lambda=380\ nm}^{\lambda=740\ nm} \Phi_{e,\lambda}\, V(\lambda) d\lambda$$

In this equation, $\Phi_{e,\lambda}$ is the spectral flux, or radiant flux per unit wavelength. The constant K_{cd} is defined in the SI standard (International System of Units) as follows: The luminous efficacy of monochromatic radiation of frequency 540 × 10^{12} Hz, K_{cd}, is 683 lm/W. This frequency corresponds to the wavelength 555.17 nm (peak of sensitivity of the eye in day vision), as

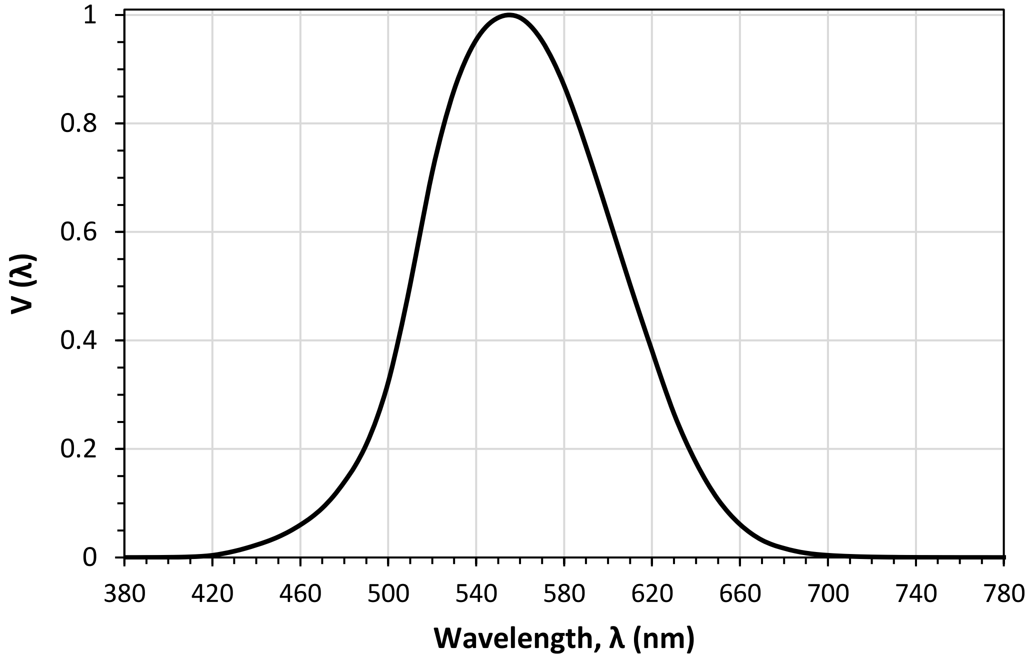

***Figure 1**: The 2° standard observer spectral luminous efficiency function for photopic vision, CIE 1924 V(λ) function.*

λ (nm)	V(λ)	λ (nm)	V(λ)	λ (nm)	V(λ)	λ (nm)	V(λ)
380	0.00003900	*480*	*0.13902000*	*580*	*0.87000000*	*680*	*0.01700000*
385	0.00006400	485	0.16930000	585	0.81630000	685	0.01192000
390	0.00012000	490	0.20802000	590	0.75700000	690	0.00821000
395	0.00021700	495	0.25860000	595	0.69490000	695	0.00572300
400	0.00039600	500	0.32300000	600	0.63100000	700	0.00410200
405	0.00064000	505	0.40730000	605	0.56680000	705	0.00292900
410	0.00121000	510	0.50300000	610	0.50300000	710	0.00209100
415	0.00218000	515	0.60820000	615	0.44120000	715	0.00148400
420	0.00400000	520	0.71000000	620	0.38100000	720	0.00104700
425	0.00730000	525	0.79320000	625	0.32100000	725	0.00074000
430	0.01160000	530	0.86200000	630	0.26500000	730	0.00052000
435	0.01684000	535	0.91485010	635	0.21700000	735	0.00036110
440	0.02300000	540	0.95400000	640	0.17500000	740	0.00024920
445	0.02980000	545	0.98030000	645	0.13820000	745	0.00017190
450	0.03800000	550	0.99495010	650	0.10700000	750	0.00012000
455	0.04800000	555	1.00000000	655	0.08160000	755	0.00008480
460	0.06000000	560	0.99500000	660	0.06100000	760	0.00006000
465	0.07390000	565	0.97860000	665	0.04458000	765	0.00004240
470	0.09098000	570	0.95200000	670	0.03200000	770	0.00003000
475	0.11260000	575	0.91540000	675	0.02320000	775	0.00002120
480	0.13902000	580	0.87000000	680	0.01700000	780	0.00001499

***Table 1**: The 2° standard observer spectral luminous efficiency function for photopic vision, CIE 1924 V(λ) function.*

calculated from the speed of light in vacuum, c = 299 792 458 m/s. It is noteworthy that the integration is only carried out over the wavelengths for which $V(\lambda) \neq 0$. Since V(λ) is given by a table of empirical values (see Figure 1 and Table 1), it is best to complete the integration numerically. The typical range for this calculation is $380 \text{ nm} \leq \lambda \leq 740 \text{ nm}$.

This short incursion into photometry highlights the fundamental role played by V(λ). We will not discuss photometry further as it falls outside the scope of this technical note. The interested readers can find details on this topic in the Technical Guide "*A general guide to high-efficiency LED exposure systems*" released by idonus in 2019 (DOI: 10.13140/RG.2.2.29958.32323).

We also temporarily leave the photopic function V(λ), to which we will return in section 5.

3. Wright and Guild's colour matching experiments

In the late 1920's, William David Wright (Imperial College, London) and John Guild (National Physical Laboratory, Teddington) conducted a series of experiments aimed at consolidating the emerging theory of colorimetry. Both lived in the UK, and the studies they carried out were similar in many aspects, but they completed their work independently from each other. This is an important point because it was the concordance between their respective findings that motivated the CIE to define a standardised colour space based on their results.

In short, observers were asked to match monochromatic lights by mixing three primary-colour lights. Using the data collected from only 17 observers (10 for Wright and 7 for Guild), Wright and Guild could quantitatively measure how the monochromatic colours of the spectrum can be reproduced when beams of red, green, and blue light are added. The combination of their data, gathered by Guild in 1931, forms the basis of the CIE colour matching functions that are still in use in today's colorimetry science. Below, we describe the basic parts of Guild's experimental setup. The main difference from Wright's experiments lied in the wavelengths chosen for the primaries.

- **Monochromator** – The source of light (reference white) was a 40-watt automobile headlight lamp, the filament of which was focused by means of an achromatic lens and a prism on the slit of a constant deviation monochromator.
- **Primaries** – The 3 primaries were obtained by passing the light from an opal-bulb gas-filled lamp through red, green, and blue gelatine filters. The wavelength chosen for these primaries were 700 nm (red), 546.1 nm (green), and 435.8 nm (blue). The choice of very specific wavelengths for green and blue lied in the availability of filters specially designed to isolate these spectral lines prominent in mercury. As for the red stimulus, because the hue was found to be constant for wavelengths higher than about 670 nm, a convenient procedure was to use a very deep red filter.
- **Trichromatic colorimeter** – The observer was facing a square field of view divided into two rectangular portions by a horizontal line. The size of the aperture was such that the square subtended an angle of approximately 2° at the observer's eye (hence, the name "2-degree standard observer"). The test field contained the monochromatic colour to be matched. The other, the matching field, contained a blend of the primaries which could be adjusted by means of three control handles, their amounts being measured on graduations attached to the shutters. If a test colour couldn't be matched, the observer could instead introduce one of the primaries into the test field (*i.e.*, subtracted).

With this system, any monochromatic colour (C_λ) could be uniquely represented by a colour equation of the form:

$$C_\lambda = rR + gG + bB, \qquad r + g + b = 1,$$

where *r*, *g*, and *b* are the trichromatic coefficients, positive or negative, representing certain proportions of the red (R), green (G), and blue (B) primaries, respectively[1]. In Figure 2, we show a graph of the trichromatic coefficients plotted from the values adopted by the CIE Committee on Colorimetry in 1931. Since these coefficients are normalized, the CIE *rg* chromaticity space can also be represented in the form of the *rg* chromaticity diagram shown in Figure 3, the *b* coordinate being directly deduced from the *r* and *g* coordinates. The curved line is the spectral locus. The red (1,0,0), green (0,1,0), and blue (0,0,0) points on that curve correspond to the 3 primary colours. The white point (E) is the Equal Energy point where $r = g = b$. Notice that the spectral locus is completely enclosed in a triangle (dashed lines) formed by the virtual points C_r, C_g and C_b. The Equal Energy point and the vertices of the dashed triangle are located at the following coordinates:

	E	C_r	C_g	C_b
r	1/3	1.27496	-1.73930	-0.74310
g	1/3	-0.27777	2.76726	0.14091
b	1/3	0.00281	-0.02795	1.60219

***Table 2**: Coordinates of the equal energy point and three primary colours in the CIE RGB colour space.*

To draw the dashed triangle in Figure 3, the vertices were chosen arbitrarily but conscientiously. The line that passes through C_b and C_r is known as the alychne, a word coined by Erwin Schrödinger, meaning "no light" in Ancient Greek: it is

[1] In CIE's convention, lower case letters represent the chromaticity coordinates in a colour space of the same name written in upper case letters (*e.g.*, *r*, *g*, and *b* coordinates in the CIE RGB colour space).

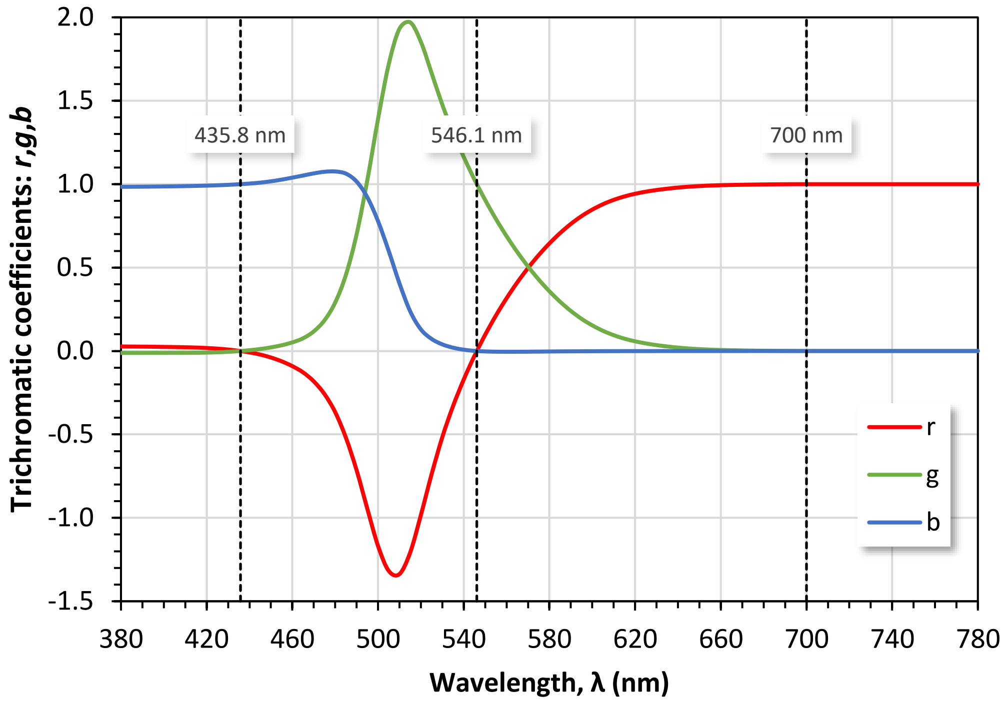

Figure 2: Trichromatic coefficients r,g, and b of the CIE 1931 standard observer.

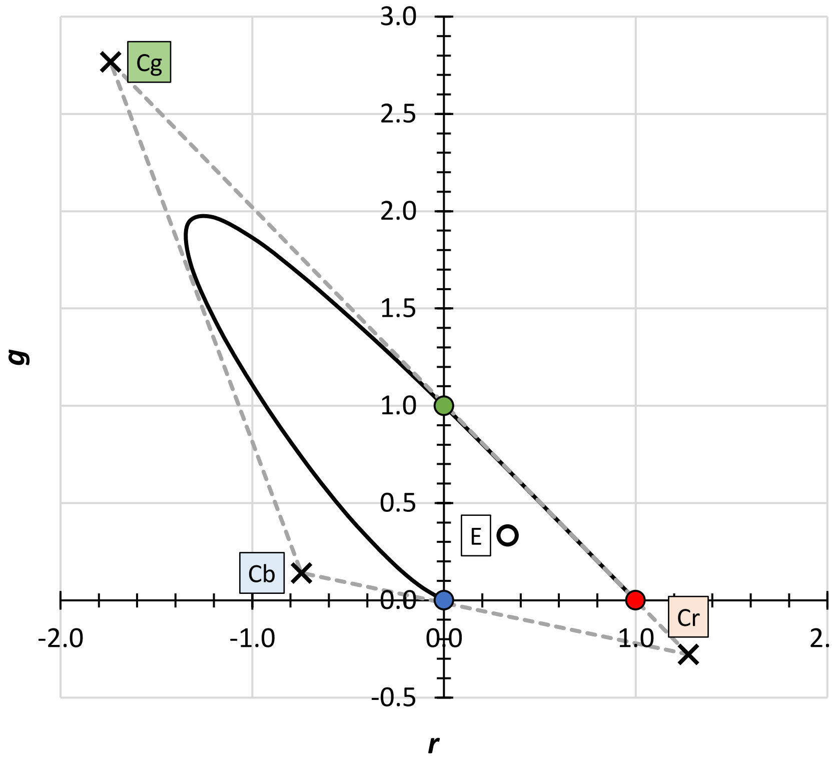

Figure 3: CIE 1931 rg chromaticity diagram.

the line of zero luminance in the colour space. The line that passes through C_g and C_r is located just above the green and red primaries. As for the line that passes through C_g and C_b, it was chosen so as to keep the size of the triangle small. On the spectral locus, the closest point is approximately that of the 504 nm wavelength.

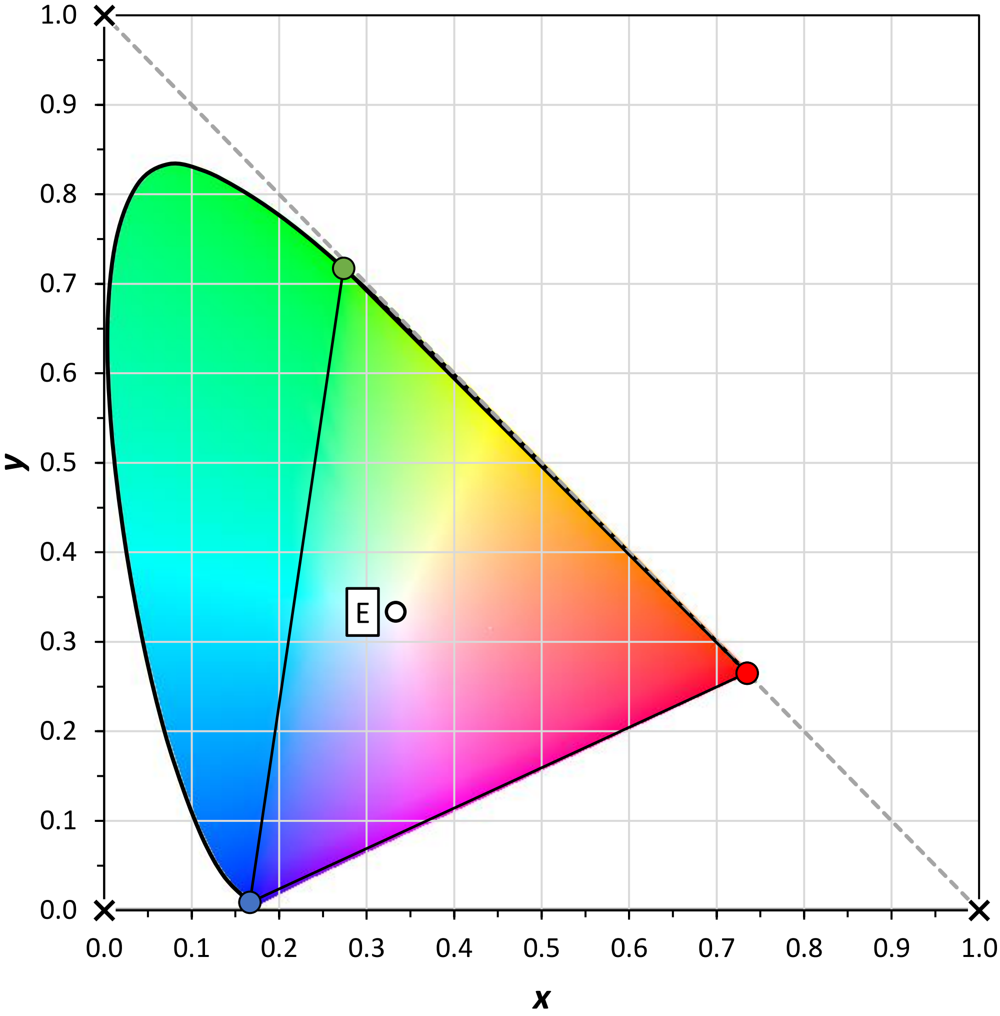

***Figure 4**: CIE 1931 xy chromaticity diagram. The triangle defined by the red, green, and blue points is the gamut of the original RGB space.*

4. CIE XYZ colour space

The closed area formed by the spectral locus curve and the alychne line encompasses all the colours visible for the average human eye. In Figure 3, the right-angled triangle formed by the red, green, and blue points only contains the colours that can be synthesized with this particular choice of trichromatic primaries (*i.e.*, 435.8 nm, 546.1, and 700 nm colour stimuli). To get free from a particular choice of the primary colours, the CIE opted for a virtual XYZ colour space built from the virtual primaries C_r, C_g and C_b. These primaries have the following coordinates in the CIE XYZ colour space: C_r (1,0,0), C_g (0,1,0), C_b (0,0,1). For anyone familiar with linear algebra, an immediate matrix transformation that comes to mind is that obtained from the trichromatic coordinates given in Table 1:

$$M_0 = \begin{bmatrix} 1.27496 & -1.73930 & -0.74310 \\ -0.27777 & 2.76726 & 0.14091 \\ 0.00281 & -0.02795 & 1.60219 \end{bmatrix}^{-1} = \begin{bmatrix} 0.90878 & 0.57494 & 0.37093 \\ 0.09122 & 0.41876 & 0.00548 \\ 0.00000 & 0.00630 & 0.62359 \end{bmatrix}$$

However, this transformation would translate the Equal Energy point to the following *xyz* coordinates:

$$E_0 = \begin{bmatrix} E_{0,x} \\ E_{0,y} \\ E_{0,z} \end{bmatrix} \begin{bmatrix} 0.61822 \\ 0.17182 \\ 0.20996 \end{bmatrix}$$

The CIE wanted the coordinate of the Equal Energy point to remain unchanged through the transformation RGB → XYZ. The *xyz* coordinates being normalized, one can see that the appropriate matrix transformation, *M*, is of the form:

$$M = M_{RGB\to XYZ} = k \cdot \frac{1}{3}\begin{bmatrix} 1/E_{0,x} & 0 & 0 \\ 0 & 1/E_{0,y} & 0 \\ 0 & 0 & 1/E_{0,z} \end{bmatrix} M_0 = k \begin{bmatrix} 0.49000 & 0.31000 & 0.20000 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00000 & 0.01000 & 0.99000 \end{bmatrix}$$

$$M = \begin{bmatrix} 2.768892 & 1.751748 & 1.130160 \\ 1 & 4.5907 & 0.0601 \\ 0 & 0.056508 & 5.594292 \end{bmatrix}$$

where *k* is a scaling factor that doesn't affect the *xyz* coordinates (*k* = 5.6508, see section 5 for explanations). In this equation, the 1/3 factor is a simple rewriting of the coordinates of the point of Equal Energy. The result of this transformation is shown in Figure 4. In this CIE XYZ chromaticity diagram, we have added colours to represent all the colours that can be seen by the human eye. As stated above, only the colours that are inside the triangle formed by the red, green, and blue can be synthesized with this choice of trichromatic primaries. This triangle represents the **gamut** of this colour space: the subset of colours which can be accurately represented with these primaries. Since most monitors and printers are designed to fulfil the standardized sRGB colour space, which has a smaller gamut than the CIE RGB colour space, part of the colours inside the triangle cannot be reproduced accurately. This remark is even more true for the colours outside the gamut triangle: they can simply not be reproduced. For this reason, the blue-cyan-green colours shown in the upper part of the graph are simply an extension of the colours calculated on the edge of the triangle.

5. CIE colour matching functions

So far, we have only dealt with normalized coefficients (*rgb* and *xyz*) that provide the chromaticity coordinates in a colour space. In its effort to standardize the XYZ colour space, the CIE introduced absolute colour matching functions from which all the normalized coefficients are calculated. They are written with an overbar: $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ for the XYZ space and $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ for the RGB space. Owing to the fundamental role played by the photopic function, the CIE imposed that $\bar{y}(\lambda)$ be the photopic function $V(\lambda)$ introduced in section 2:

$$\bar{y}(\lambda) = V(\lambda)$$

It can be straightforwardly shown that the other colour matching function must satisfy the following relations:

$$\bar{x}(\lambda) = \frac{x}{y} V(\lambda), \qquad \bar{z}(\lambda) = \frac{V(\lambda)}{y}(1 - x - y)$$

As for the $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ colour matching functions, because they have a physical significance, an appropriate scaling factor needed to be used for XYZ→RGB transformation. Indeed, the $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ functions were normalized so that:

$$\bar{r}(\lambda) + 4.5907\,\bar{g}(\lambda) + 0.0601\,\bar{b}(\lambda) = V(\lambda)$$

The ratios 1 : 4.5907 : 0.0601 correspond to the relative luminances of the trichromatic stimuli. Hence, the parameter *k* = 5.6508 that was used above without further explanation (1 + 4.5907 + 0.0601 = 5.6508).

The colour matching functions $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, $\bar{b}(\lambda)$ can be obtained through the matrix transformation:

$$M_{XYZ\to RGB} = M^{-1} = \begin{bmatrix} 2.768892 & 1.751748 & 1.130160 \\ 1 & 4.5907 & 0.0601 \\ 0 & 0.056508 & 5.594292 \end{bmatrix}^{-1} \approx \begin{bmatrix} 0.418456 & -0.158657 & -0.082833 \\ -0.091167 & 0.252426 & 0.015707 \\ 0.000921 & -0.002550 & 0.178595 \end{bmatrix}$$

The colour matching functions for the XYZ and RGB spaces are shown in Figure 5 and Figure 6, respectively.

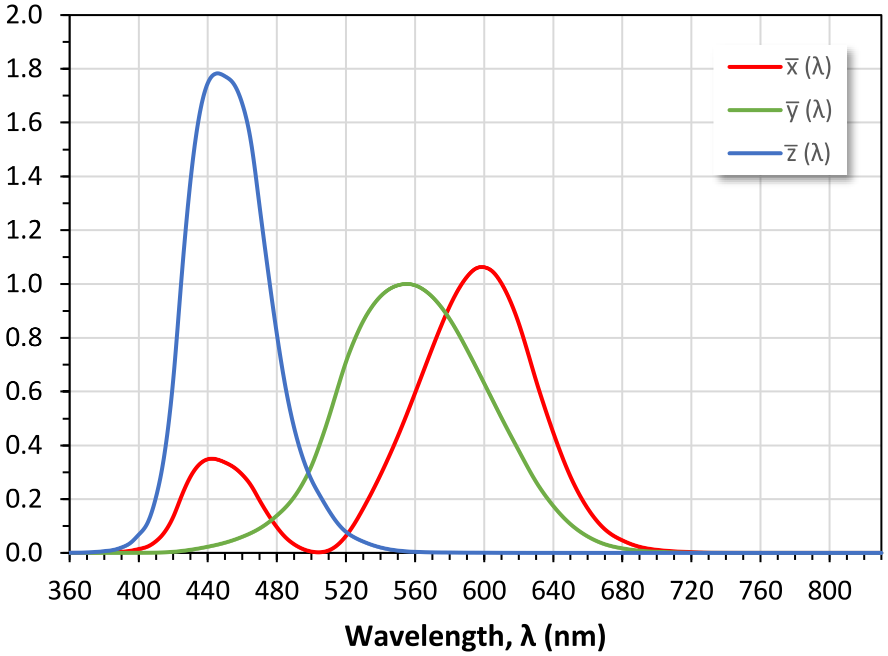

Figure 5*: CIE XYZ colour matching functions.*

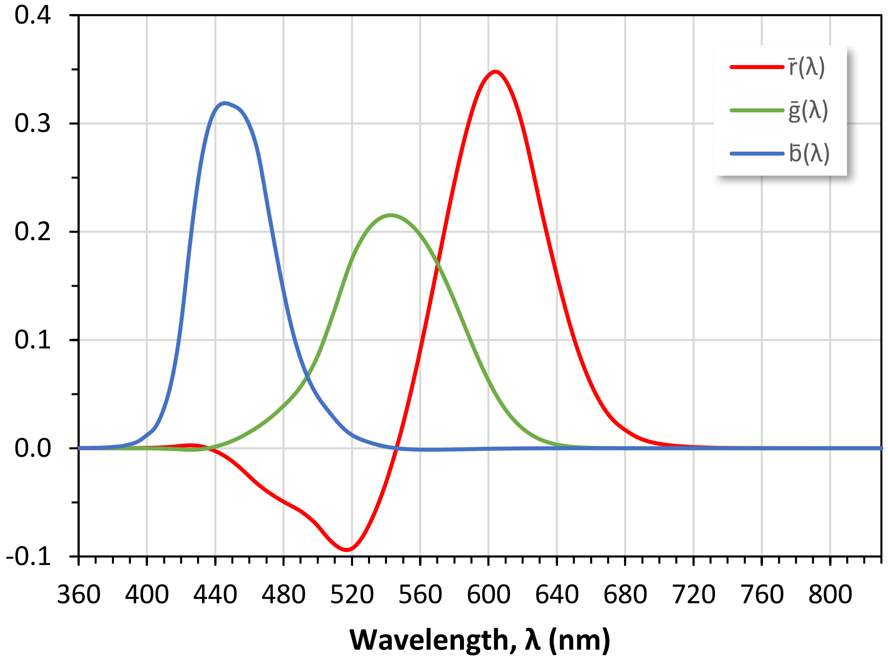

Figure 6*: CIE RGB colour matching functions*

6. sRGB colour space and Planckian locus

We have mentioned above the sRGB colour space which is widely used for consumer electronic devices (displays, digital cameras, printers), and which is also the standard for the Internet. Conversion from CIE XYZ to sRGB requires a scaling so that the white point is D65. The illuminant D65 is a CIE standard that is intended to represent average daylight. It was calculated from the average spectral power distribution of the Sun and has a correlated colour temperature of approximately 6500 K.

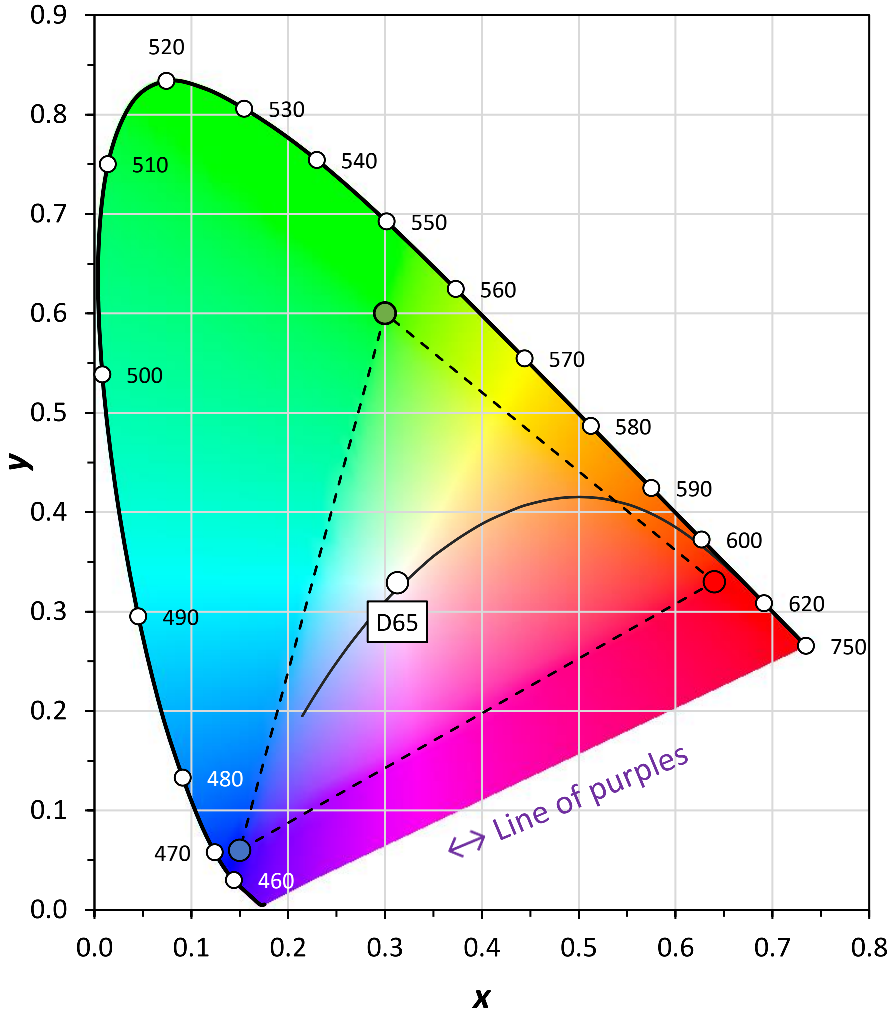

Figure 7: CIE 1931 chromaticity diagram and sRGB colour gamut. The curve passing through the white point D65 is the Planckian locus. The points highlighted on the spectral locus correspond to monochromatic lights emitted at the indicated wavelengths (in nm).

The coordinates of D65 in the CIE XYZ colour space can be approximated using the theoretical spectral radiant exitance of a black body heated at T = 6503.15 K:

$$M_{e,\lambda} = \frac{2\pi hc^2}{\lambda^5}\left(e^{\frac{hc}{kT}} - 1\right)^{-1}$$

where *h* is the Planck constant, *c* is the speed of light in vacuum, *k* is the Boltzmann constant, *T* is the temperature in Kelvin, λ is the wavelength. In practice, this equation is used to draw the Planckian locus curve. In Figure 7, we have drawn the CIE 1931 XYZ chromaticity diagram and included the Planckian locus curve. The dashed triangle shows the gamut of the sRGB colour space. The colours were calculated using the XYZ→sRGB transformation matrix from the official sRGB documentation:

$$M_{XYZ\rightarrow sRGB} = \begin{bmatrix} 3.2406255 & -1.537208 & -0.4986286 \\ -0.9689307 & 1.8757561 & 0.0415175 \\ 0.0557101 & -0.2040211 & 1.0569959 \end{bmatrix}$$

The GNU Octave script prepared to generate the CIE diagram of Figure 7 is provided at the end of this technical note (3 pages, grey background)[2].

[2] GNU Octave, scientific programming language. Version used: GNU Octave 6.2.0.
[https://octave.org/]

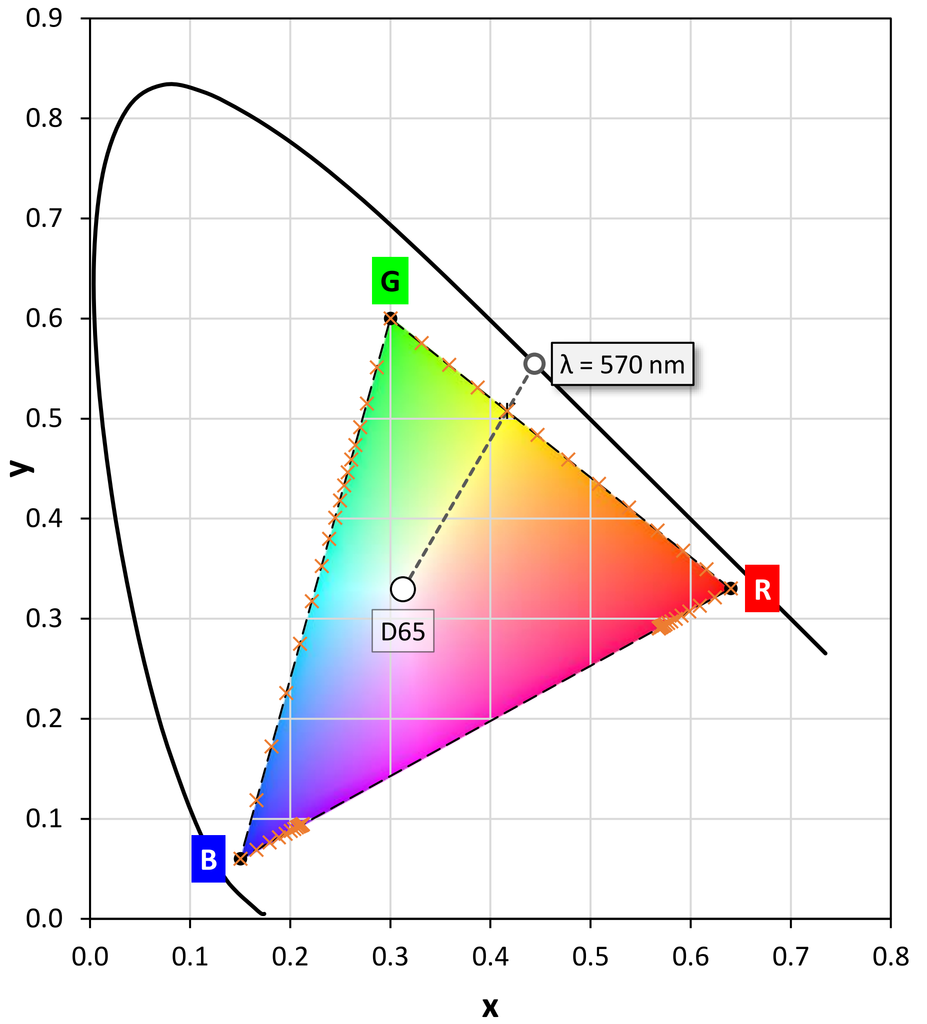

***Figure 8**: Calculating saturated colour coordinates for sRGB colour space. The line that joins point D65 and a specific point on the spectral locus (e.g., 570 nm monochromatic wavelength) crosses the triangle gamut at a specific coordinate (e.g., x = 0.4166; y = 0.5074).*

7. Rendering of the colour spectrum in the sRGB colour space

In Figure 7, the points on the spectral locus curve represent pure monochromatic colours visible to the average human eye. However, only the points inside the dashed triangle can be represented in the sRGB colour space. Taking this limitation into account, the best way to perform optimum rendering of the colour spectrum is to use the corresponding saturated values that are found on the edges of the gamut triangle. An example is shown in Figure 8 for 570 nm monochromatic colour (yellow). The calculation must be made using the reference white colour (for sRGB colour space, the white point is D65). The saturated colour is found straightforwardly using the line that passes through the white point and a given point on the spectral locus: It crosses the gamut triangle at a well-defined coordinate near the spectral locus. These calculated coordinates are plotted in Figure 9. Using the XYZ→sRGB transformation matrix introduced in section 6, one can easily obtain the curve plotted in Figure 10. Notice that the procedure followed is identical to the one described in section 5: $\bar{x}$ (λ), $\bar{y}$ (λ), $\bar{z}$ (λ) must be calculated prior to completing the transformation.

These linear RGB values are not the final result. Gamma correction must be applied to obtain the final non-linear sRGB values. The following formula transforms the linear values into sRGB:

$$C_{sRGB} = \begin{cases} 12.92\, C_{linear}, & C_{linear} \leq 0.0031308 \\ 1.055 C_{linear}^{1/2.4} - 0.055, & C_{linear} \leq 0.0031308 \end{cases}$$

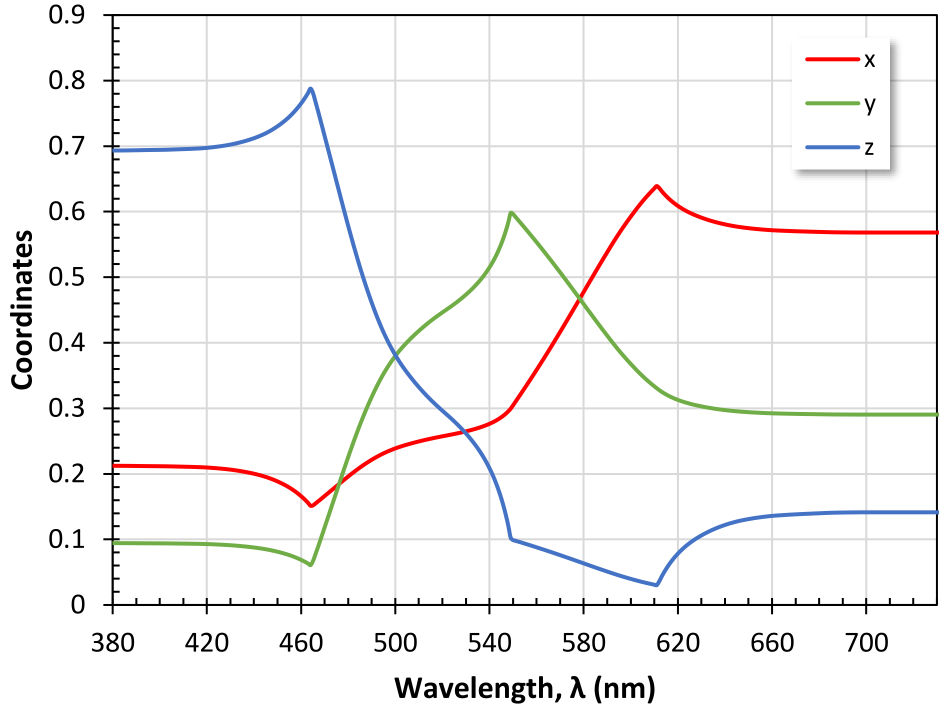

Figure 9: Saturated colour coordinates in sRGB colour space and corresponding monochromatic wavelengths. For example, for 570 nm (yellow), the corresponding coordinates are: x = 0.4166; y = 0.5074; z = 0.076.

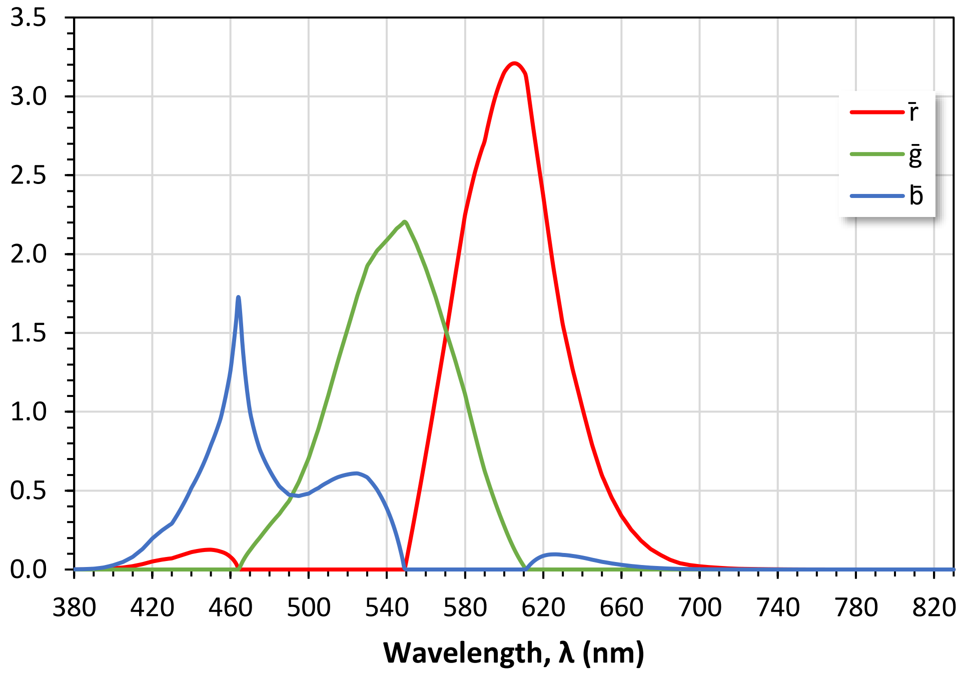

Figure 10: linear RGB values and corresponding wavelengths for colour coordinates shown in Figure 8.

where C is either R, G, or B. These gamma-corrected values are usually clipped to the 0 to 1 range. For 24-bit RGB, 8 bits (256 levels) are used for each colour channel. This is done by multiplying C_{sRGB} by 255 and rounding the result to an integer.

The clipping creates discontinuities that would result in sharp transition points in the colour spectrum. To smoothen the resulting colour spectrum, it is advisable to perform a 2D convolution of the data (*e.g.*, conv2 function in Matlab or Octave). The final result is shown in Figure 11 and the GNU Octave script prepared to generate this graph is provided at the end of this technical note (1 page, blue background). Considering the limited gamut of the sRGB colour space, the spectral colour plot obtained with this method is globally satisfying.[3]

8. Summary

Colour spectrum representation was chosen as an example to illustrate the usefulness of the CIE 1931 chromaticity diagram. As you could expect, the use of the CIE 1931 XY colour space extends far beyond this simple case. To take but only few applications, colour LEDs data sheets are always provided with their bin specification in terms of xy coordinates in the CIE XY colour space; lightings are specified with a colour rendering index (CRI) calculated in the CIE colour space by comparing the colour rendering of a test source with an ideal source with a given correlated colour temperature (CCT); comparison of colour cameras passes through gamut analysis that is best displayed using the CIE chromaticity diagram.

[3] Useful reading in relation with section 7: "Making Color Spectrum Plots – Part 2," Steve Eddins, July 20, 2020.
[https://blogs.mathworks.com/steve/2020/07/20/making-color-spectrum-plots-part-2/]

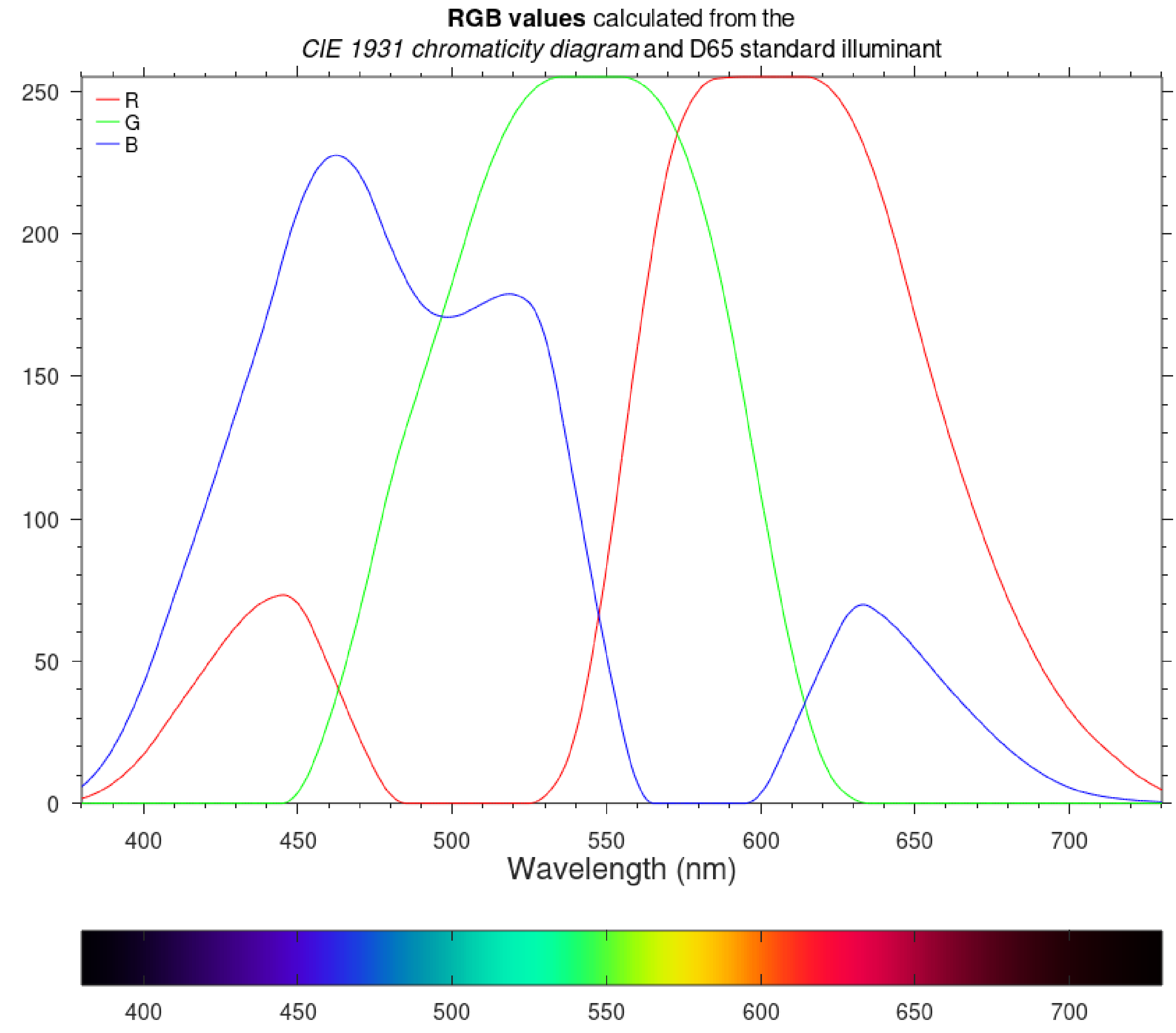

Figure 11: *Rendering of the solar spectrum with consideration of the sRGB gamut limitations.*
Plotted with GNU Octave v. 6.2.0 using the script 'RGB_to_spectrum.m' provided in this technical note.

```
% Colour representation of the CIE 1931 chromaticity diagram
% Save the file as, e.g., 'CIE_1931_chromaticity_diagram.m'. Run this script in GNU Octave (v. 5.2.0).
% Prepared by: Christophe Yamahata, idonus Sàrl, June 2022    |    Adapted from:
%           https://ch.mathworks.com/matlabcentral/fileexchange/40640-computational-colour-science-using-matlab-2e

clear;

% Chromaticity coordinates of spectrum locus + line of purples
CIE_x = [ 0.17411 0.17380 0.17141 0.16888 0.16441 0.16111 0.15664 0.15099 0.14396 0.13550 0.12412 ...
0.10960 0.09129 0.06871 0.04539 0.04076 0.03620 0.03176 0.02749 0.02346 0.01970 0.01627 ...
0.01318 0.01048 0.00817 0.00628 0.00487 0.00398 0.00364 0.00386 0.00464 0.00601 0.00799 ...
0.01060 0.01387 0.01777 0.02224 0.02727 0.03282 0.03885 0.04533 0.05218 0.05932 0.06672 ...
0.07430 0.08205 0.08994 0.09794 0.10602 0.11416 0.12235 0.13055 0.13870 0.14677 0.15472 ...
0.16253 0.17024 0.17785 0.18539 0.19288 0.20031 0.20769 0.21503 0.22234 0.22962 0.23689 ...
0.24413 0.25136 0.25858 0.26578 0.27296 0.28013 0.28729 0.29445 0.30160 0.30876 0.31592 ...
0.32306 0.33021 0.33736 0.34451 0.35167 0.35881 0.36596 0.37310 0.38024 0.38738 0.39451 ...
0.40163 0.40873 0.41583 0.42292 0.42999 0.43704 0.44406 0.45106 0.45804 0.46499 0.47190 ...
0.47878 0.48561 0.49241 0.49915 0.50585 0.51249 0.51907 0.52560 0.53207 0.53846 0.54479 ...
0.55103 0.55719 0.56327 0.56926 0.57515 0.58094 0.58665 0.59222 0.59766 0.60293 0.60803 ...
0.61298 0.61778 0.62246 0.62704 0.64823 0.66576 0.68008 0.69151 0.70792 0.71903 0.72599 ...
0.73342 0.73469 0.67858 0.62248 0.56639 0.51030 0.45421 0.39811 0.34202 0.28593 0.22984 0.17411 ]';

CIE_y = [ 0.00496 0.00492 0.00510 0.00690 0.01086 0.01379 0.01771 0.02274 0.02970 0.03988 0.05780 ...
0.08684 0.13270 0.20072 0.29498 0.31698 0.33990 0.36360 0.38792 0.41270 0.43776 0.46295 ...
0.48821 0.51340 0.53842 0.56307 0.58712 0.61045 0.63301 0.65482 0.67590 0.69612 0.71534 ...
0.73341 0.75019 0.76561 0.77963 0.79211 0.80293 0.81202 0.81939 0.82516 0.82943 0.83227 ...
0.83380 0.83409 0.83329 0.83159 0.82918 0.82621 0.82277 0.81893 0.81478 0.81040 0.80586 ...
0.80124 0.79652 0.79169 0.78673 0.78163 0.77640 0.77105 0.76559 0.76002 0.75433 0.74852 ...
0.74262 0.73661 0.73051 0.72432 0.71806 0.71172 0.70532 0.69884 0.69231 0.68571 0.67906 ...
0.67237 0.66563 0.65885 0.65203 0.64517 0.63829 0.63138 0.62445 0.61750 0.61054 0.60357 ...
0.59659 0.58961 0.58262 0.57563 0.56865 0.56167 0.55472 0.54777 0.54084 0.53393 0.52705 ...
0.52020 0.51339 0.50661 0.49989 0.49321 0.48659 0.48003 0.47353 0.46709 0.46073 0.45443 ...
0.44823 0.44210 0.43606 0.43010 0.42423 0.41846 0.41276 0.40719 0.40176 0.39650 0.39141 ...
0.38648 0.38171 0.37705 0.37249 0.35140 0.33401 0.31975 0.30834 0.29203 0.28094 0.27401 ...
0.26658 0.26531 0.23948 0.21363 0.18778 0.16192 0.13607 0.11022 0.08437 0.05852 0.03266 0.00496 ]';


% CIE standard illuminant D65. For sRGB, the white point has chromaticity coordinates (0.3127, 0.3290).
% This white point is also known as D65, which is an estimation of the white colour produced by mid-day sunlight.
D65 = [0.31271 0.32902];

% Planckian locus: black body locus
planckian_locus_x = [ 0.7347 0.7214 0.7092 0.6956 0.6813 0.6670 0.6528 0.6388 0.6250 0.6116 ...
0.5985 0.5857 0.5732 0.5650 0.5611 0.5496 0.5269 0.5054 0.4857 0.4677 0.4514 0.4366 0.4053 ...
0.3805 0.3605 0.3439 0.3083 0.2997 0.2854 0.2651 0.2513 0.2413 0.2338 0.2279 0.2232 0.2146 ]';
planckian_locus_y = [ 0.2653 0.2786 0.2907 0.3041 0.3179 0.3315 0.3445 0.3565 0.3675 0.3772 ...
0.3858 0.3931 0.3993 0.4027 0.4043 0.4081 0.4133 0.4152 0.4147 0.4123 0.4087 0.4042 0.3909 ...
0.3767 0.3633 0.3507 0.3186 0.3096 0.2938 0.2690 0.2506 0.2365 0.2253 0.2163 0.2089 0.1950  ]';

% sRGB gamut (colour triangle defined by the RGB primaries)
% XYZ to sRGB transformation matrix
M = [  3.2406255  -1.537208   -0.4986286;
      -0.9689307   1.8757561   0.0415175;
       0.0557101  -0.2040211   1.0569959 ];

%%% End of Part 1/3
```

```
% Gamut: colour triangle defined by the RGB primaries
Gamut =[inv(M)(:,1)/sum(inv(M)(:,1)) inv(M)(:,2)/sum(inv(M)(:,2)) inv(M)(:,3)/sum(inv(M)(:,3))];
gamut_x = [Gamut(1,:) Gamut(1,1)];
gamut_y = [Gamut(2,:) Gamut(2,1)];

function [rgb] = xy2srgb(xy, M)

  xyz = [xy(:,1) xy(:,2) 1-xy(:,1)-xy(:,2)];
  rgb = xyz*M';

  % Get brighter spectral colours, including a good yellow, scale up the linear RGB values,
  % allowing them to get higher than 1.0. Then, for each colour, scale all components back down,
  % if necessary, so that the maximum component value is 1.0.
  rgb = rgb / max(rgb(:));
  rgb = rgb*2.5;
  S = max(rgb,[],2);
  S = max(S,1);
  rgb = rgb ./ S;
  rgb = min(max(rgb,0),1);

  % Convert to nonlinear RGB values for the final result
  if(rgb <= 0.0031308)
    rgb = 12.92*rgb;
  else
    rgb = 1.055*rgb.^(1/2.4)-0.055;
  endif

end

N = length(CIE_x); i = 1; e = D65;
White_point = e;
sectors_mono = 8;          % The larger these values, the better the graph quality
sectors_purple = 7;        % (e.g., > 5 sectors gives good results).
sectors = sectors_mono;
xy4rgb = zeros(N*sectors*4,5,'double');
for w = 1:N                              % equiv. wavelength
  w2 = mod(w,N)+1;
  a1 = atan2(CIE_y(w) -e(2),CIE_x(w) -e(1));          % start angle
  a2 = atan2(CIE_y(w2) -e(2),CIE_x(w2) -e(1));          % end angle
  r1 = ((CIE_x(w) - e(1))^2 + (CIE_y(w) - e(2))^2)^0.5;  % start radius
  r2 = ((CIE_x(w2) - e(1))^2 + (CIE_y(w2) - e(2))^2)^0.5;  % end radius
  if(w >= N-10 || w <=10)
    sectors = sectors_purple;
  else
    sectors = sectors_mono;
  endif

for c = 1:sectors                    % colourfulness (saturation)
    % patch polygon
    xy(1,1) = e(1)+r1*cos(a1)*c/sectors;
    xy(1,2) = e(2)+r1*sin(a1)*c/sectors;
    xy(2,1) = e(1)+r1*cos(a1)*(c-1)/sectors;

%%% End of Part 2/3
```

```
        xy(2,2) = e(2)+r1*sin(a1)*(c-1)/sectors;
        xy(3,1) = e(1)+r2*cos(a2)*(c-1)/sectors;
        xy(3,2) = e(2)+r2*sin(a2)*(c-1)/sectors;
        xy(4,1) = e(1)+r2*cos(a2)*c/sectors;
        xy(4,2) = e(2)+r2*sin(a2)*c/sectors;

        % Compute RGB for vertices
        rgb = xy2srgb(xy, M);
        % Store the results
        xy4rgb(i:i+3,1:2) = xy(:,1:2);
        xy4rgb(i:i+3,3:5) = rgb;
        i = i + 4;
    end
end

% Draw the CIE chromaticity diagram
min_x = 0; min_y = 0; max_x = 0.8; max_y = 0.9;
title_graph = 'CIE 1931 Chromaticity Diagram';

figure('Name', title_graph,'NumberTitle','off', 'ToolBar', 'none', 'Position', [600 50 750 900]);
plot( [min_x; max_x], [0; 0], 'k-', 'LineWidth', 1.2, ...
        [0; 0], [min_y; max_y], 'k-', 'LineWidth', 1.2, [0; 0.8], [1; 0.2], 'k--', 'LineWidth', 1);

% Will disappear progressively. We will plot it again at the end.
hold on
plot( White_point(1),White_point(2), 'o', 'MarkerFaceColor', 'black', 'MarkerSize',12, CIE_x, CIE_y,'k-', 'LineWidth', 3);

text(0.1,-0.17,'To interrupt this script, open the Command Window tab in {\itOctave} and type {\bfCTRL + C}', 'Color',
[0.5 0.5 0.5],'FontSize',12);

set(gca,'FontSize',18); set(gca,'TickDir','out'); set(gca, 'XMinorTick', 'on', 'YMinorTick', 'on');
axis equal; axis([min_x max_x min_y max_y]);
xticks(0:0.1:max_x); yticks(0:0.1:max_y);
xlabel('x','FontSize',26); ylabel('y','FontSize',26);
title(title_graph,'FontSize',28);
[rows cols] = size(xy4rgb);
f = [1 2 3 4]; v = zeros(4,3,'double');
for i = 1:4:rows
    v(:,1:2) = xy4rgb(i:i+3,1:2);
    patch('Vertices',v, 'Faces',f, 'EdgeColor','none', 'FaceVertexCData',xy4rgb(i:i+3,3:5),'FaceColor','interp');
    refreshdata; drawnow;
end

% Last layer: graphs on top of the coloured area
hold on
plot( planckian_locus_x, planckian_locus_y, 'Color', 0.6*[1 1 1],'LineWidth',2, ...
      White_point(1),White_point(2), 'o', 'MarkerFaceColor', 'black', 'MarkerSize',12, ...
      White_point(1),White_point(2), 'o', 'MarkerFaceColor', 'white', 'MarkerSize',8, ...
      gamut_x, gamut_y, 'k-', 'LineWidth', 1, CIE_x, CIE_y,'k-', 'LineWidth', 3);

%%% End of Part 3/3
```

```
%% This GNU Octave (v. 6.2.0) script was prepared by Christophe Yamahata, idonus Sàrl.
%% Copy the script and save the file as, e.g., 'RGB_to_spectrum.m'

clear;
lambda = [ 380:1:730]'; % Wavelength range: 380 nm - 730 nm (1 nm increment)
R = [ 1 1 1 2 2 2 2 2 3 3 3 4 5 7 8 9 10 11 13 14 15 17 18 20 21 23 25 26 28 29 31 33 35 37 39 41 43 45 47 49 51 52 ...
53 54 55 56 57 58 59 60 61 63 64 66 67 69 70 72 73 75 76 77 77 78 78 79 79 80 80 81 81 79 78 76 75 73 71 70 68 67 ...
65 59 52 46 39 33 26 20 13 7 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 5 11 16 22 27 32 38 43 49 54 67 80 93 106 119 132 145 158 171 ...
184 191 197 204 211 218 224 231 238 244 251 251 252 252 253 253 253 254 254 255 255 255 255 255 255 255 255 ...
255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 ...
255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 251 247 243 239 235 230 226 222 218 214 ...
209 205 200 196 191 186 182 177 173 168 164 160 157 153 149 145 141 138 134 130 127 123 120 117 114 110 107 ...
104 100 97 94 92 99 86 84 81 78 75 73 70 68 65 63 60 58 55 53 50 48 45 44 42 41 39 38 37 35 34 32 31 30 29 28 27 ...
26 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 10 9 9 8 8 7 7 6 6 5 ]';
G = [ 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 8 16 24 32 40 48 56 64 72 80 84 88 92 96 100 103 107 111 115 119 122 124 127 ...
129 132 135 137 140 142 145 149 152 156 159 163 167 170 174 177 181 185 189 193 197 201 205 209 213 217 221 ...
224 228 231 235 238 241 245 248 252 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 ...
255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 255 ...
255 255 255 255 255 255 252 248 245 242 239 235 232 229 225 222 217 212 207 202 197 192 187 182 177 172 167 ...
161 156 150 145 140 134 129 123 118 110 101 93 84 76 68 59 51 42 34 31 27 24 20 17 14 10 7 3 0 0 0 0 0 0 0 0 0 0 0 ...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ]';
B = [ 3 4 5 6 7 8 8 9 10 11 12 15 17 20 23 26 28 31 34 36 39 42 45 49 52 55 58 61 65 68 71 75 79 82 86 90 94 98 101 ...
105 109 111 114 116 118 121 123 125 127 130 132 136 140 144 148 152 155 159 163 167 171 175 178 182 185 189 ...
193 196 200 203 207 212 217 221 226 231 236 241 245 250 255 253 250 248 245 243 241 238 236 233 231 227 222 ...
218 213 209 205 200 196 191 187 185 183 180 178 176 174 172 169 167 165 165 165 165 165 166 166 166 166 166 ...
166 167 168 169 170 172 173 174 175 176 177 178 178 179 180 181 181 182 183 183 184 184 183 183 183 183 182 ...
182 182 181 181 178 175 172 169 166 162 159 156 153 150 135 120 105 90 75 60 45 30 15 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 7 15 22 29 37 44 51 58 66 73 73 ...
74 74 74 75 75 75 75 76 76 75 75 74 73 73 72 71 70 70 69 67 66 64 63 61 60 58 57 55 54 53 51 50 49 48 46 45 44 42 ...
41 40 39 37 36 35 34 33 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 9 8 8 7 7 6 6 5 5 4 4 4 3 ...
3 3 3 3 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 ]';
RGB = [ R G B];

% First, perform a convolution to smoothen the data
RGB = conv2(RGB, ones(31,1)/31,'same');

% Draw the graph
figure('NumberTitle','off', 'Name', "Rendering of the solar spectrum with consideration of the sRGB gamut limitations",
'ToolBar', 'none', 'MenuBar', 'none', 'Position', [100 200 1000 800]);
plot(lambda(:),RGB(:,1), 'r', lambda(:),RGB(:,2), 'g',lambda(:),RGB(:,3), 'b');
set(gca,'FontSize',16,'TickDir','out', 'XMinorTick', 'on', 'YMinorTick', 'on');
title ({'RGB values \rm{calculated from the}';
'\rm{\it{CIE 1931 chromaticity diagram}} \rm{and D65 standard illuminant}'});
legend('R','G','B', 'location', 'northwest', 'box','off');
axis([380 730 0 255]); xticks(400:50:720); yticks(0:50:250); xlabel('Wavelength (nm)','FontSize',22);

% Draw the spectrum
intensity_R=RGB(:,1)/255; intensity_G=RGB(:,2)/255; intensity_B=RGB(:,3)/255;
colormap ([intensity_R,intensity_G,intensity_B]); caxis([380 730]);
h = colorbar ("southoutside"); set(h,'fontsize',16);
```

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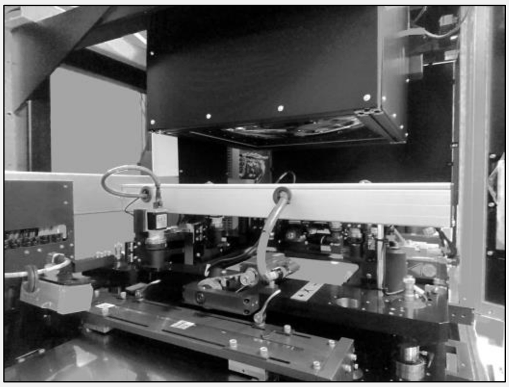
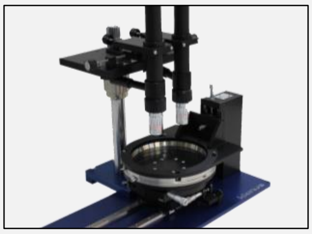
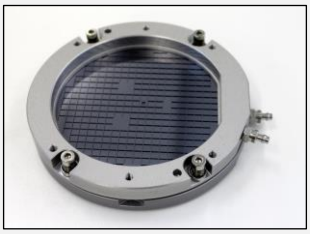


Customized UV-LED exposure system installed on a roll-to-roll machine



Double image microscope



A model of wafer chuck

About idonus

Founded in 2004, idonus is a Swiss company that develops and manufactures special equipment for the MEMS and watchmaking industries. Our product portfolio includes UV-LED exposure systems for photolithography, IR microscope for wafer inspection, vapor phase chemical etcher for silicon-based devices. Since 2016, we also provide ion implantation services and machines for the surface treatment of materials.

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